

Bianchi Type-I Model with Cosmic String in the Presence of a Magnetic Field: Spinor Description

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Abstract A Bianchi type-I cosmological model in the presence of a magnetic flux along a cosmic string is investigated. A nonlinear spinor field is used to simulate the cosmological cloud of strings. It is shown that the spinor field simulation offer the possibility to solve the system of Einstein's equation without any additional assumptions. It is pointed out that the present model is nonsingular at the end of the evolution and does not allow the anisotropic Universe to turn into an isotropic one.

Keywords Bianchi type-I model · Cosmological string · Magnetic field · Nonlinear spinor field

1 Introduction

Though the present day Universe is well described by an isotropic and homogeneous Friedmann-Robertson-Walker (FRW) model, there are serious theoretical arguments about the existence of an anisotropic phase in the evolution of the Universe [1]. These ideas were further supported by the observational data from COBE (Cosmic Background Explorer) and WMAP (The Wilkinson Microwave Anisotropy Probe) where small anisotropy in the microwave background radiation was found. These lead many cosmologists to consider the anisotropic model to describe the initial phase of the evolution which eventually decays into an isotropic FRW one.

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After the famous paper by A. Guth [2], scalar field was hugely used in simulating different cosmological models. But the question occurs, if other fields can contribute to the evolution of the Universe. As an alternative the spinor field was used due to its sensitivity to the gravitational one [3–10]. The nonlinear spinor field proved to be able to describe different cosmological models. For example it was shown that a suitable choice of nonlinearity (i) accelerates the isotropization process, (ii) gives rise to a singularity-free Universe and (iii) generates late time acceleration.

Moreover, it is shown that a nonlinear spinor field can be used to simulate a perfect fluid from ekpyrotic matter to phantom matter [11–13].

At the same time the string cosmological models have been used in attempts to describe the early Universe and to investigate anisotropic dark energy component including a coupling between dark energy and a perfect fluid (dark matter) [14, 15]. Cosmic strings are one dimensional topological defects associated with spontaneous symmetry breaking in gauge theories. Their presence in the early Universe can be justified in the frame of grand unified theories (GUT).

On the other hand, the magnetic field has an important role at the cosmological scale and is present in galactic and intergalactic spaces. Any theoretical study of cosmological models which contain a magnetic field must take into account that the corresponding Universes are necessarily anisotropic. Among the anisotropic spacetimes, Bianchi type-I space (BI) seems to be the most convenient for testing different cosmological models.

The object of this paper is to investigate a BI string cosmological model in the presence of a magnetic flux. For this purpose we use the nonlinear spinor field simulation as was described recently in a number of papers [11–13].

2 Basic Equations

We study the evolution of the Universe in presence of a cosmic string and a magnetic flux in the framework of a BI anisotropic cosmological model. For a BI spacetime the line element is given by

$$ds^2 = (dt)^2 - a_1^2(t)(dx^1)^2 - a_2^2(t)(dx^2)^2 - a_3^2(t)(dx^3)^2. \tag{1}$$

There are three scale factors a_i ($i = 1, 2, 3$) which are functions of time t only and consequently three expansion rates. In principle all these scale factors could be different and it is useful to express the mean expansion rate in terms of the average Hubble rate:

$$H = \frac{1}{3} \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \right), \tag{2}$$

where over-dot means differentiation with respect to t .

Einstein’s gravitational field equations, corresponding to the metric (1) have the form

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} = \kappa T_1^1, \tag{3a}$$

$$\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1} = \kappa T_2^2, \tag{3b}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} = \kappa T_3^3, \tag{3c}$$

$$\frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1} = \kappa T_0^0, \tag{3d}$$

where κ is the gravitational constant. The energy momentum tensor for a system of cosmic string and magnetic field in a comoving coordinate is given by

$$T_{\mu}^{\nu} = \rho u_{\mu} u^{\nu} - \lambda x_{\mu} x^{\nu} + E_{\mu}^{\nu}, \tag{4}$$

where ρ is the rest energy density of strings with massive particles attached to them and can be expressed as $\rho = \rho_p + \lambda$, where ρ_p is the rest energy density of the particles attached to the strings and λ is the tension density of the system of strings [16–18] which may be positive or negative. Here u_i is the four velocity and x_i is the direction of the string, obeying the relations

$$u_i u^i = -x_i x^i = 1, \quad u_i x^i = 0. \tag{5}$$

In (4) $E_{\mu\nu}$ is the electromagnetic field [19] and in what follows we shall choose the string and also the magnetic field along x^1 direction. In our model the electromagnetic field tensor $F^{\alpha\beta}$ has only one non-vanishing component, namely

$$F_{23} = h, \tag{6}$$

where h is presumed to be constant. For the electromagnetic field E_{μ}^{ν} one gets the following non-trivial components

$$E_0^0 = E_1^1 = -E_2^2 = -E_3^3 = \frac{h^2}{2\bar{\mu}a_2^2a_3^2} \equiv \frac{1}{2} \frac{\beta^2}{(a_2a_3)^2}, \tag{7}$$

where $\bar{\mu}$ is a constant characteristic of the medium and called the magnetic permeability. Typically $\bar{\mu}$ differs from unity only by a few parts in 10^{-5} ($\bar{\mu} > 1$ for paramagnetic substances and $\bar{\mu} < 1$ for diamagnetic). To simplify the notation, we include in a constant β the value of the electromagnetic field, h , and the magnetic permeability, $\bar{\mu}$.

Using comoving coordinates we have the following components of energy momentum tensor [20]:

$$T_0^0 - \rho = T_1^1 - \lambda = -T_2^2 = -T_3^3 = \frac{\beta^2 a_1^2}{2 \tau^2}, \tag{8}$$

where we introduce the volume scale of the BI space-time [4]

$$\tau = \sqrt{-g} = a_1 a_2 a_3, \tag{9}$$

which is connected with the Hubble rate (2), namely $\frac{\dot{\tau}}{\tau} = 3H$.

In view of $T_2^2 = T_3^3$ from (8) one finds

$$a_2 = a_3 D \exp\left(X \int \frac{dt'}{\tau}\right), \tag{10}$$

with D and X some integration constants.

On the other hand, summation of Einstein equations (3a), (3b), (3c) and three times (3d) gives:

$$\frac{\ddot{\tau}}{\tau} = \frac{1}{2} \kappa \left(3\rho + \lambda + \beta^2 \frac{a_1^2}{\tau^2} \right). \tag{11}$$

Taking into account the conservation of the energy-momentum tensor, i.e., $T_{\mu;\nu}^{\nu} = 0$, after a little manipulation of (8) one obtains [21, 22]:

$$\dot{\rho} + \frac{\dot{\tau}}{\tau}\rho - \frac{\dot{a}_1}{a_1}\lambda = 0. \tag{12}$$

It is customary to assume a relation between ρ and λ in accordance with the state equations for strings. The simplest one is a proportionality relation [16]:

$$\rho = \alpha\lambda. \tag{13}$$

Among the most usual choices of the constant α we mention the following:

$$\alpha = \begin{cases} 1, & \text{geometric string} \\ 1 + \omega, & \omega \geq 0, \text{ } p \text{ string or Takabayasi string} \\ -1, & \text{Reddy string} \end{cases} \tag{14}$$

It is possible to consider a more general barotropic relation

$$\rho = \rho(\lambda). \tag{15}$$

than the linear relation (13), subject to the restrictions imposed by the energy conditions. The weak energy condition as well as the strong one require $\rho \geq \lambda$ with $\lambda \geq 0$ or $\rho \geq 0$ with $\lambda < 0$ and the dominant energy condition implies $\rho \geq 0$ and $\rho^2 \geq \lambda^2$ [16].

3 Spinor Field Approach

Recently it was shown that a nonlinear spinor can be used to simulate different types of perfect fluids including those called ekpyrotic, phantom matter and dark energy [11–13]. Here we show that it is possible to describe cosmic strings in terms of spinor fields as well.

3.1 Spinor Field Simulation

We shall simulate the cloud formed by massive cosmic strings with particles attached along their extensions with a nonlinear spinor field described by the Lagrangian:

$$L_{sp} = \frac{i}{2} [\bar{\psi}\gamma^{\mu}\nabla_{\mu}\psi - \nabla_{\mu}\bar{\psi}\gamma^{\mu}\psi] - m\bar{\psi}\psi + F, \tag{16}$$

with F being some arbitrary function of the scalar $S = \bar{\psi}\psi$. For the simulation of the present cosmological model in which the anisotropic scale factors a_i are functions solely of time it is adequate to assume that the spinor field depends on t only. Then the corresponding components of energy-momentum tensor take the form

$$T_0^0 = mS - F, \tag{17a}$$

$$T_i^i = S \frac{dF}{dS} - F, \quad i = 1, 2, 3. \tag{17b}$$

In what follows we describe the energy density ρ of the string by T_0^0 and the tension density λ by T_1^1 in agreement with (8). Inserting (17a) and (17b) into (13) we find

$$S \frac{dF}{dS} - \left(1 - \frac{1}{\alpha}\right)F - \frac{m}{\alpha}S = 0, \tag{18}$$

with the solution

$$F = C_1 S^{(\alpha-1)/\alpha} + mS. \tag{19}$$

Here C_1 is an integration constant. The positivity of T_0^0 imposes some restriction on C_1 , namely $C_1 \leq 0$. Setting $C_1 = -\nu$ we find the cosmic string can be described by the spinor field Lagrangian

$$L_{sp} = \frac{i}{2} [\bar{\psi} \gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma^\mu \psi] - \nu S^{(\alpha-1)/\alpha}, \tag{20}$$

remarking the disappearance of the mass term [11].

With these preparatives we get

$$\rho = T_0^0 = \nu S^{(\alpha-1)/\alpha}, \tag{21a}$$

$$\lambda = T_1^1 = \frac{\nu}{\alpha} S^{(\alpha-1)/\alpha}. \tag{21b}$$

Variation on (16) with respect to ψ and $\bar{\psi}$ gives spinor field equations with nonlinear terms. On the other hand from the spinor field equations for S one finds [4]

$$\dot{S} + \frac{\dot{\tau}}{\tau} S = 0, \tag{22}$$

with the solution

$$S = \frac{C_0}{\tau}, \tag{23}$$

C_0 being a constant.

Taking into account this simple behavior of S we have finally

$$\rho = R \tau^{-\frac{\alpha-1}{\alpha}}, \tag{24a}$$

$$\lambda = \frac{R}{\alpha} \tau^{-\frac{\alpha-1}{\alpha}}, \tag{24b}$$

with the constant $R = \nu C_0^{\frac{\alpha-1}{\alpha}}$.

Using the above formulas for ρ and λ , from (12) we can determine the anisotropic factor a_1 :

$$a_1 = A_1 \tau, \tag{25}$$

A_1 being a constant of integration. On the other hand, from (9) and (10) we obtain

$$a_2 = \sqrt{\frac{D}{A_1}} \exp\left(\frac{X}{2} \int \frac{dt'}{\tau}\right), \tag{26}$$

and

$$a_3 = \frac{1}{\sqrt{A_1 D}} \exp\left(-\frac{X}{2} \int \frac{dt'}{\tau}\right). \tag{27}$$

3.2 Asymptotic Behavior

In what follows, we study the equation for τ in details. Using the above expressions for ρ, λ, a_1 we get from (11):

$$\ddot{\tau} = \frac{\kappa}{2} \frac{3\alpha + 1}{\alpha} R \tau^{\frac{1}{\alpha}} + \frac{\kappa \beta^2 A_1^2}{2} \tau. \tag{28}$$

We can evaluate the derivative of τ with respect to t which leads finally to a solution in quadrature

$$\int \frac{d\tau}{\sqrt{[\kappa R(3\alpha + 1)/(\alpha + 1)]\tau^{(\alpha+1)/\alpha} + [\kappa \beta^2 A_1^2/2]\tau^2 + C}} = t + t_0, \tag{29}$$

C and t_0 being some integration constants.

In spite of the fact that this equation cannot be explicitly solved, the asymptotic behavior of the solutions for $t \rightarrow \infty$ could be found.

3.2.1 Case I

In most cases, (29) provides a standing expansion of the volume scale of the BI spacetime for t growing.

Indeed, for

$$\frac{\alpha + 1}{\alpha} \leq 2, \tag{30}$$

i.e. $\alpha \geq 1$ or $\alpha < 0$, the term with τ^2 at the denominator of the l.h.s. of (29) is dominant and we get an exponential behavior

$$\tau \propto \exp t, \tag{31}$$

for large t . In this case, only the anisotropic factor a_1 presents an exponential increase for $t \rightarrow \infty$, while the scale factors a_2 and a_3 tend to constants.

Concerning the asymptotic behavior of ρ and λ we infer from (24a), (24b) that they tend to zero as

$$\rho, \lambda \propto \frac{1}{\exp(\frac{\alpha-1}{\alpha}t)}. \tag{32}$$

3.2.2 Case II

For $0 < \alpha < 1$, there is no solution with a $\tau \rightarrow \infty$ behavior for $t \rightarrow \infty$. In this case from (29) one finds:

$$\tau^{\frac{\alpha-1}{2\alpha}} \propto t.$$

Since $\alpha - 1 < 0$, τ cannot tend to infinity as $t \rightarrow \infty$. Therefore the model does not admit a consistent solution for $0 < \alpha < 1$ in agreement with the discussion of the general barotropic equation (15) from Sect. 2.

Taking into account the absence of consistent solutions for $0 < \alpha < 1$ as it was shown above, for the present spinor simulation, in what follows we shall refer only to the situations described in Sect. 3.2.1 (Case I). It is worth also mentioning that the present model does not support in any case a vanishing of τ for $t \rightarrow \infty$.

3.3 Numerical Simulations

In this subsection we graphically illustrate the evolution of energy density ρ , volume scale τ and metric functions a_1, a_2 and a_3 . In doing so we rewrite the equation for τ (28) in terms of the Hubble parameter H and introduce a new function T :

$$\dot{\tau} = 3H\tau, \tag{33a}$$

$$\dot{H} = -3H^2 + \frac{\kappa}{6} \frac{3\alpha + 1}{\alpha} R\tau^{\frac{1-\alpha}{\alpha}} + \frac{\kappa\beta^2 A_1^2}{6}, \tag{33b}$$

$$\dot{T} = \frac{T}{\tau}. \tag{33c}$$

We also rewrite the metric functions in terms of T , which now read

$$a_1 = A_1\tau, \quad a_2 = \sqrt{\frac{D}{A_1}} T^{X/2}, \quad a_3 = \sqrt{\frac{1}{A_1 D}} T^{-X/2}. \tag{34}$$

Let us now numerically solve the system (33). In doing so for simplicity we set $\kappa = 1, A_1 = 1, D = 1$ and $X = 1$. In Fig. 1 we plot the evolution of energy density ρ for different values of α , namely for $\alpha = -1$ and $\alpha = 1.5$. Evolution of τ corresponding to these values

Fig. 1 View of energy density of the cosmic string for different value of α

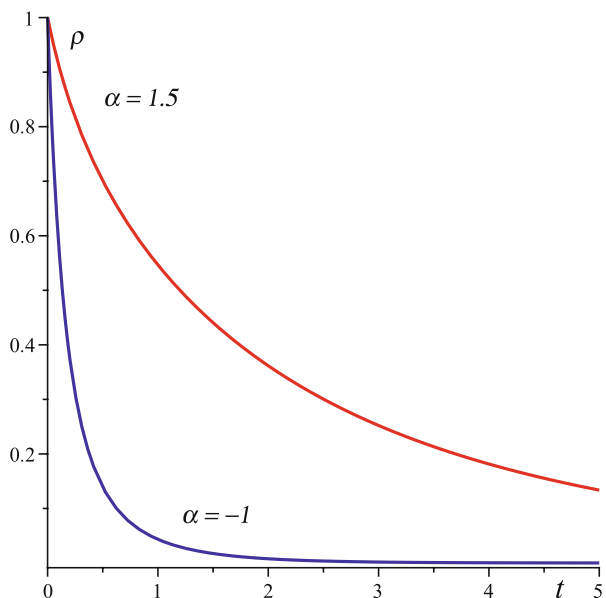


Fig. 2 Evolution of the Universe corresponding to the energy densities given in Fig. 1

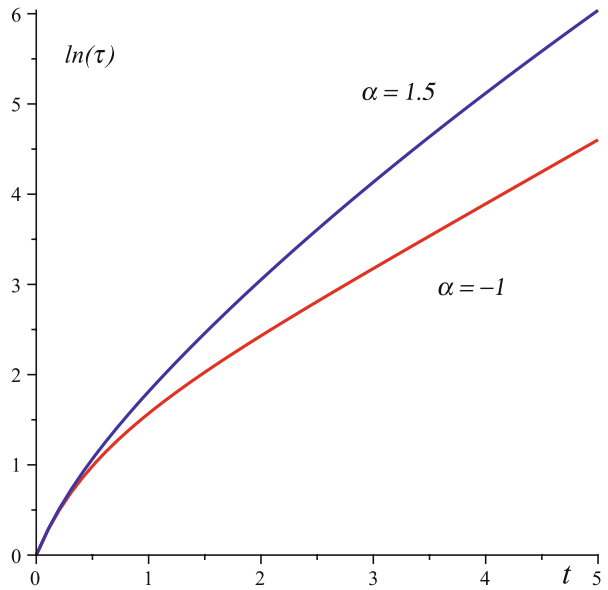
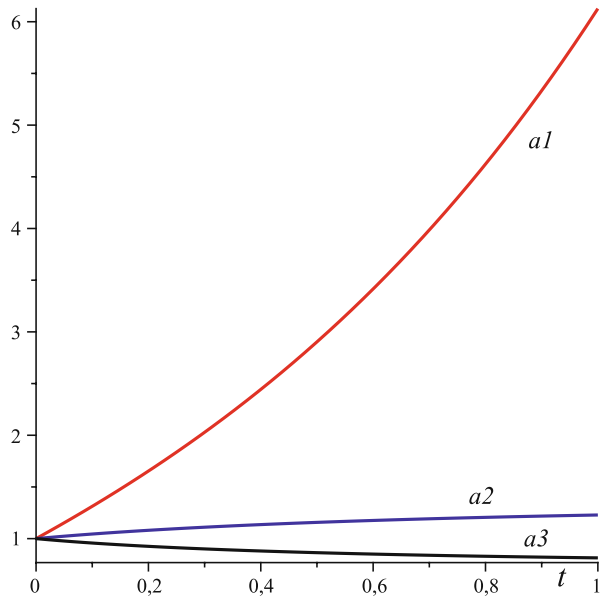


Fig. 3 Evolution of metric functions for $\alpha = 1.5$

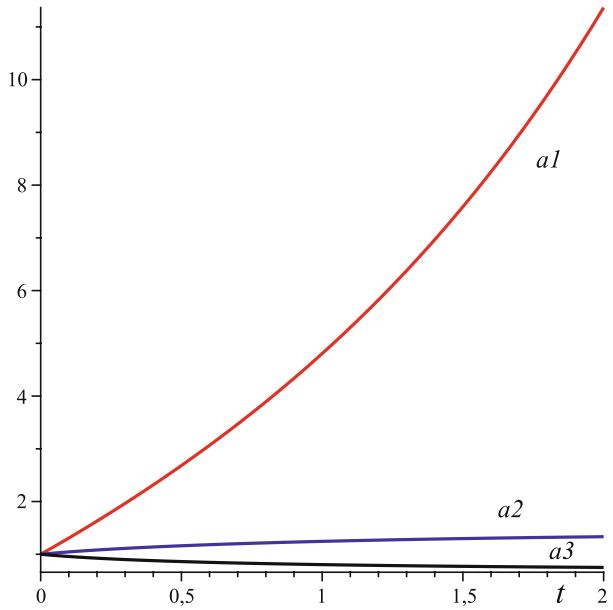


of parameters in shown in Fig. 2. For simplicity we also set the initial values of τ , H and T to be unity.

The numerical simulations support the behavior described in (31), (32). From Fig. 2 one finds at first, in the case of a negative α the Universe expands slower than it does as for a positive α , though in both cases we have an exponential growth for large time.

In Figs. 3 and 4 we plot the behavior of metric functions for positive and negative values of α . As is seen from the figures, with the expansion of the Universe a_1 increases exponen-

Fig. 4 Evolution of metric functions for $\alpha = -1$



tially, while a_2, a_3 tend to constants. Thus we see that introduction of cosmic string does not allow isotropization of initially anisotropic space-time. In the following subsections we discuss the singularity problem and isotropization process in detail.

3.4 Singularities

The next task is to investigate the space-time singularities in the present model. Let us rewrite the metric functions in the following form:

$$a_i = C_i \tau^{N_i} \exp\left(Y_i \int \frac{dt'}{\tau}\right), \tag{35}$$

where $C_1 = A_1$, $C_2 = \sqrt{D/A_1}$, $C_3 = 1/\sqrt{DA_1}$, $N_1 = +1, N_2 = N_3 = 0, Y_1 = 0, Y_2 = X/2$, and $Y_3 = -X/2$. Then the first and the second derivatives of the metric functions take the form:

$$\frac{\dot{a}_i}{a_i} = N_i \frac{\dot{\tau}}{\tau} + \frac{Y_i}{\tau}, \tag{36a}$$

$$\frac{\ddot{a}_i}{a_i} = N_i \frac{\ddot{\tau}}{\tau} + (N_i^2 - N_i) \left(\frac{\dot{\tau}}{\tau}\right)^2 + (2N_i - 1) Y_i \frac{\dot{\tau}}{\tau^2} + \frac{Y_i^2}{\tau^2}. \tag{36b}$$

We study the singularities analyzing the regularity properties of the Kretschmann scalar which for the metric (1) reads

$$\mathcal{K} = 4 \left[\left(\frac{\ddot{a}_1}{a_1}\right)^2 + \left(\frac{\ddot{a}_2}{a_2}\right)^2 + \left(\frac{\ddot{a}_3}{a_3}\right)^2 + \left(\frac{\dot{a}_1}{a_1} \frac{\dot{a}_2}{a_2}\right)^2 + \left(\frac{\dot{a}_2}{a_2} \frac{\dot{a}_3}{a_3}\right)^2 + \left(\frac{\dot{a}_3}{a_3} \frac{\dot{a}_1}{a_1}\right)^2 \right]. \tag{37}$$

Evidently, a singularity can occur when some or all scale factors a_i tend to zero or infinity. We shall follow the criteria given in [23] which states:

- (i) t finite, some $a_i \rightarrow \infty$. If at least one scale factor becomes infinite at finite t , it is a curvature singularity.
- (ii) t finite, some $a_i \rightarrow 0$. If more than one scale factor turns to zero at finite t , it is a singularity. If only one scale factor is zero at finite t , the spacetime can be nonsingular.
- (iii) $t \rightarrow \infty$, some $a_i \rightarrow \infty$. Such an asymptotic can only be singular if at least one scale factor grows faster than exponentially:

$$a_i(t) \gg \exp(k|t|), \quad k = \text{const.} > 0.$$

- (iv) $t \rightarrow \infty$, some $a_i \rightarrow 0$. Such an asymptotic can only be singular if at least one scale factor vanishes faster than exponentially:

$$a_i(t) \ll \mathcal{O}(\exp[-k|t|]), \quad k = \text{const.} > 0.$$

Looking at the asymptotic behaviors described above (Sect. 3.2) we conclude that in the spinor field approach of the cosmic strings the present model is nonsingular at the end of the evolution. No scale factor of the BI metric presents a growth faster than exponentially or vanishing faster than exponentially for $t \rightarrow \infty$.

3.5 Isotropization

Since the present-day Universe is surprisingly isotropic, it is important to see whether our anisotropic BI model evolves into an isotropic FRW model. Isotropization means that at large physical times t , when the volume factor τ tends to infinity, the three scale factors $a_i(t)$ grow at the same rate. Two wide-spread criteria of isotropization read

$$\mathcal{A} = \frac{1}{3} \sum_{i=1}^3 \frac{H_i^2}{H^2} - 1 \rightarrow 0, \tag{38a}$$

$$\Sigma^2 = \frac{1}{2} \mathcal{A} H^2 \rightarrow 0. \tag{38b}$$

Here \mathcal{A} and Σ^2 are the average anisotropy and shear, respectively, while $H_i = \dot{a}_i/a_i$ is the directional Hubble parameter evaluated in (36a). We also investigate the isotropization condition proposed in [23]

$$\frac{a_i}{a} \Big|_{t \rightarrow \infty} \rightarrow \text{const}, \tag{39}$$

where $a(t) = \tau^{1/3}$ is the average scale factor. Provided that condition (39) is fulfilled, by rescaling some of the coordinates, we can make $a_i/a \rightarrow 1$, and the metric will become manifestly isotropic at large t .

In order to study the isotropization process in the present model, we evaluate from (35)

$$\frac{a_i}{a} = \frac{a_i}{\tau^{1/3}} = C_i \tau^{N_i - 1/3} \exp\left(Y_i \int \frac{dt'}{\tau}\right). \tag{40}$$

Taking into account the value of N_i and Y_i we see that a_i/a do not tend to constants as $t \rightarrow \infty$. For example, in the generic case with the exponential expansion of the volume of the Universe (31), $a_1/a \rightarrow \infty$, $a_2/a \rightarrow 0$ and $a_3/a \rightarrow 0$. So in the case considered, no isotropization process takes place. Indeed, at the early stage of evolution, where $\tau \rightarrow 0$ we

assume $a_1 \rightarrow \infty$, $a_2 \rightarrow 0$ and $a_3 \rightarrow 0$, that is at this stage the Universe looks like a one-dimensional string. In the asymptotic region where $t \rightarrow \infty$ and $\tau \rightarrow \infty$ we have $a_1 \rightarrow \infty$, while a_2 and a_3 evaluating to finite values.

Let us also examine the possibility of an isotropization process using conditions (38). In view of (36) from (38) in the generic case (31) one finds:

$$\mathcal{A} = 2, \quad (41a)$$

$$\Sigma^2 = \frac{1}{9}. \quad (41b)$$

As one sees, both \mathcal{A} and Σ^2 are some positive defined, non vanishing quantities. Thus, in general, the present model does not undergo an isotropization process, the anisotropic feature of the Universe from the early stages being retained.

4 Conclusions

We have studied the evolution of a anisotropic universe given by a BI cosmological model in presence of cosmic strings and magnetic field. In doing so we exploit the spinor simulation of cosmic strings that allows us to solve the system of Einstein's equations. Exact solutions have been supplemented with some numerical evaluations. It is shown that the presence of cosmic strings does not allow the anisotropic Universe to evolve into an isotropic one.

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References

- Misner, C.W.: *Astrophys. J.* **151**, 431 (1968)
- Guth, A.: *Phys. Rev. D* **23**, 347 (1981)
- Henneaux, M.: *Phys. Rev. D* **21**, 857 (1980)
- Saha, B.: *Phys. Rev. D* **64**, 123501 (2001)
- Armendáriz-Picón, C., Greene, P.B.: *Gen. Relativ. Gravit.* **35**, 1637 (2003)
- Saha, B., Boyadjiev, T.: *Phys. Rev. D* **69**, 124010 (2004)
- Ribas, V.O., Devecchi, F.P., Kremer, G.M.: *Phys. Rev. D* **72**, 123502 (2005)
- Saha, B.: *Phys. Part. Nucl.* **37**(1), S13 (2006)
- Saha, B.: *Phys. Rev. D* **74**, 124030 (2006)
- Saha, B.: *Phys. Rev. D* **69**, 124006 (2004)
- Krechet, V.G., Fel'chenkov, M.L., Shikin, G.N.: *Gravit. Cosmol.* **14**, 292 (2008)
- Saha, B.: Spinor model of a perfect fluid. *Cent. Eur. J. Phys.* (2010, to appear). [arXiv:0901.1387](https://arxiv.org/abs/0901.1387) [gr-qc]
- Saha, B.: Spinor model of a perfect fluid: examples. *Rom. Rev. Phys.* (2010, to appear). [arXiv:0902.2097](https://arxiv.org/abs/0902.2097) [gr-qc]
- Koivisto, T., Mota, D.F.: *Phys. Lett. B* **644**, 104 (2007)
- Koivisto, T., Mota, D.F.: *Phys. Rev. D* **75**, 023518 (2007)
- Letelier, P.S.: *Phys. Rev. D* **28**, 2414 (1983)
- Pradhan, A., Yadav, A.K., Singh, R.P., Singh, V.K.: *Astrophys. Space Sci.* **312**, 145 (2007)
- Khadekar, G.S., Tade, S.D.: *Astrophys. Space Sci.* **310**, 47 (2007)
- Lichnerowicz, A.: *Relativistic Hydrodynamics and Magnetohydrodynamics*. Benjamin, New York (1967)
- Saha, B.: *Astrophys. Space Sci.* **299**, 149 (2005)
- Saha, B., Visinescu, M.: *Astrophys. Space Sci.* **315**, 99 (2008)
- Saha, B., Rikhvitsky, V., Visinescu, M.: *Cent. Eur. J. Phys.* **8**, 113 (2010)
- Bronnikov, K.A., Chudaeva, E.N., Shikin, G.N.: *Class. Quantum Gravity* **21**, 3389 (2004)